

# Analysis of $B_c^+ \rightarrow J/\psi a_1(1260)^+$ in perturbative QCD approach

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## Abstract

In this paper, we analyse the hadronic decay of  $B_c^+ \rightarrow J/\psi a_1(1260)^+$  in perturbative QCD approach (pQCD), where  $a_1(1260)$  is a axial-vector meson and  $J/\psi$  is a vector meson. The experimental data of the branching ratio of this decay is less than  $(1.2 \times 10^{-3})$ . We obtained that the branching ratio is  $(1.02^{+0.04+0.03+0.01}_{-0.08-0.05-0.01}) \times 10^{-3}$  and has a good agreement with the experimental result.

## 1 Introduction

The  $B_c$  meson is a ground state of  $\bar{b}c$  system and is the only heavy meson embracing two heavy quarks b and c. Because lifetime, mass and the relative Cabibbo-Kobayashi-Maskawa (CKM) matrix element between b and c quark are different, the decay rate of the two quarks is different. Therefore studying of  $B_c$  decays provide information about CP violation and standard model. Since the  $B_c$  meson carries explicit flavor, it can not annihilate via strong interaction or electromagnetic interaction like the mesons consisting of  $c\bar{c}$  or  $b\bar{b}$ . It can only decay via weak interaction that is an ideal platform to study weak decays of heavy quarks. Compared with the  $B_{u,d,s}$  mesons, the decays of the  $B_c$  meson are rather different from those of  $B$  or  $B_s$  meson, since in  $B_c$  meson both b and c can decay while the other serves as a spectator, or annihilating into pairs of leptons or light mesons. Exclusive modes containing of  $B_c \rightarrow h_1 h_2$  decays ( $h_i$  are the vector, axial-vector, pseudo-scalar, tensor and scalar). Two-body hadronic  $B_c$  decay meson has studied by many author [1, 2, 3, 4, 5, 6, 7]. In this research, the two body hadronic decay  $B_c^+ \rightarrow J/\psi a_1(1260)^+$  are studied, where  $a_1(1260)$  and  $J/\psi$  are axial-vector and vector respectively. Axial-vector mesons are mesons with the quantum numbers  $J_P = 1^+$ . In the quark model, there are two distinct types of axial-vector mesons, namely,  $^3P_1$  and  $^1P_1$ , which carry the quantum numbers  $J^{PC} = 1^{++}$  and  $J^{PC} = 1^{+-}$  respectively. Experimentally, the  $J^{PC} = 1^{++}$  nonet consists of  $a_1(1260)$ ,  $f_1(1285)$ ,  $f_1(1420)$ , and  $K_{1A}$ , while the  $J^{PC} = 1^{+-}$  nonet has  $b_1(1235)$ ,  $h_1(1170)$ ,  $h_1(1380)$  and  $K_{1B}$ . In the  $SU(3)$

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flavor limit, these mesons can not mix with each other. Due to the  $G$ -parity the chiral-even two-parton light-cone distribution amplitudes of the  $^3P_1$  ( $^1P_1$ ) mesons are symmetric (antisymmetric) under the interchange of the momentum fractions of quark and anti-quark in the  $SU(3)$  limit. The  $^3P_1$  meson behaves in a similar way to the vector meson; this is not the case for the  $^1P_1$  meson. For the latter, its decay constant vanishes in the  $SU(3)$  limit. Since  $c$  quarks mass is about one third of  $b$  quark and  $m_c/m_{B_c} \sim 0.2$ , therefore  $b$  quark in a  $B_c$  carry the large part of the energy. This assumption allows us to employ the  $k_T$  factorization theorem to the  $b$  decay in  $B_c$  meson. In this work we shall study  $B_c^+ \rightarrow J/\psi \ a_1(1260)^+$  decays in the perturbative QCD approach based on the  $k_T$  factorization[8]. We know that the light quark in  $B$  meson is soft, while it is collinear in the final state light meson, so a hard gluon is necessary to kick the light spectator quark in the  $B$  meson. By keeping the transverse momentum  $k_T$  in the quark and gluon propagators, the end point singularity in the collinear factorization can be eliminated. Because of the additional energy scale introduced by the transverse momentum, double logarithms will appear in the QCD radiative corrections. With the Sudakov resummation, we can include the leading double logarithms for all loop diagrams, in association with the soft contribution. This makes the pQCD approach more reliable and consistent. The appealing feature of the pQCD factorization is that form factors can be computed in terms of wave functions and hard kernels. Although the  $k_T$  factorization is not gauge invariant and produces an IR finite hard kernel [9], the pQCD method provides a good platform to study of  $B$  meson two-body non-leptonic decays[10]. There are three scales in the  $B$  meson non-leptonic decays  $M_W$ ,  $m_b$  and  $1/b$ . The electroweak decay of  $B$  meson is characterized by the  $W$  boson mass  $M_W$ . The second scale  $m_b$  reflects the scale of energy in the decay. Since the  $b$  quark decay scale  $m_b$  is much smaller than the electroweak scale  $M_W$ , the QCD corrections are nonnegligible. The third scale  $1/b$  is the factorization scale, with  $b$  being the conjugate variable of parton transverse momenta. Dynamics below  $1/b$  is regarded as being completely nonperturbative, and can be parametrized into a meson wave function  $\phi(x)$ . The meson wave functions are not calculable in pQCD. But they are universal, channel independent. This paper is organized as follows. In section II, we study decay constant of axial-vector, form factor, wave function and distribution amplitudes for  $B_c$ , vector and axial-vector mesons. In section III, we calculate the branching ratio and CP violation for this decay, section IV contain input quantities parameters and section V contain the main conclusion.

## 2 Theoretical framework

For the considered decay, the related weak effective Hamiltonian can be written as

$$H_{eff} = \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* (C_1(\mu) O_1 + C_2(\mu) O_2), \quad (1)$$

with the Fermi constant  $G_F$ , Cabibbo-Kobayashi-Maskawa(CKM) matrix elements  $V$  and Wilson coefficients  $C_i(\mu)$  at the renormalization scale  $\mu$ .

The local operators are written as

$$\begin{aligned} O_1 &= (\bar{q}_i u_i)_{V-A} (\bar{c}_j b_j)_{V-A}, \\ O_2 &= (\bar{q}_i u_j)_{V-A} (\bar{c}_j b_i)_{V-A}, \end{aligned} \quad (2)$$

where  $i$  and  $j$  are color indices. The operators  $O_1$  and  $O_2$  are called the current-current operators. The hard part of the pQCD approach contains six quarks rather than four quarks. This is called six-quark effective theory or six-quark operator. A  $B_c \rightarrow M_2 M_3$  decay amplitude is factorized into the convolution of the six-quark hard kernel (H), the jet function (J) and the Sudakov factor (S) with the bound-state wave functions ( $\Phi$ ) as follows,

$$A(B_c \rightarrow M_2 M_3) = \Phi_{B_c} \otimes H \otimes J \otimes S \otimes \Phi_{M_2} \otimes \Phi_{M_3}. \quad (3)$$

In the practical applications to heavy  $B_c$  meson decays, the decay amplitude of Eq. (3) in the pQCD approach can be written as,

$$A(B_c \rightarrow M_2 M_3) \sim \int d^4 k_1 d^4 k_2 d^4 k_3 \text{Tr}[\Phi_{B_c}(k_1) \Phi_{M_2}(k_2) \times \Phi_{M_3}(k_3) H(k_1, k_2, k_3, t)], \quad (4)$$

where  $k_3$ s are momenta of light quarks included in each mesons, and Tr denotes the trace over Dirac and color indices. The function  $H(k_1, k_2, k_3, t)$  describes the four quark operator and the spectator quark connected by a hard gluon whose  $q_2$  is in the order of  $\lambda_{QCD} m_{B_c}$ . The wave function  $\Phi_{B_c}(k_1)$  and  $\Phi_{M_i}$  describe the hadronization of the quark and anti-quark in the  $B_c$  meson and the final state light meson  $M_i$ .

The two final state meson momenta can be written as

$$P_1 = \frac{m_{B_c}}{\sqrt{2}}(1, 1, 0_T), \quad P_2 = \frac{m_{B_c}}{\sqrt{2}}(1 - r_3^2, r_2^2, 0_T), \quad P_3 = \frac{m_{B_c}}{\sqrt{2}}(r_3^2, 1 - r_2^2, 0_T), \quad (5)$$

where  $r_i = m_i/M_{B_c}$ . Putting the quark momenta in  $B_q$ ,  $M_2$  and  $M_3$  meson as  $K_1$ ,  $K_2$ , and  $K_3$ , respectively, we can choose

$$k_1 = (x_1 P_1^+, 0, k_{1T}), \quad k_2 = (x_2 P_2^+, 0, k_{2T}), \quad k_3 = (0, x_3 P_3^-, k_{3T}). \quad (6)$$

Then the integration over  $k_1^-$ ,  $k_2^-$ , and  $k_3^+$  will lead to the decay amplitude in the pQCD approach,

$$A(B_c \rightarrow M_1 M_2) \sim \int dx_1 dx_2 dx_3 b_1 db_1 b_2 db_2 b_3 db_3 \times \text{Tr}[C(t) \Phi_{B_c}(x_1, b_1) \Phi_{M_2}(x_2, b_2) \Phi_{M_3}(x_3, b_3)] \times H(x_i, b_i, t) S_t(x_i e^{-S(t)}], \quad (7)$$

where  $b_i$  is the conjugate space coordinate of  $k_{iT}$ , and  $t$  is the largest energy scale in function  $H(x_i, b_i, t)$ . The large logarithms  $\ln(m_W/t)$  are included in the Wilson coefficients  $C(t)$ . The large double logarithms ( $\ln^2 x_i$ ) are summed by the threshold resummation, and they lead to the jet function  $S_t(x_i)$  which smears the end-point singularities on  $x_i$  [11]. The last term,  $e^{-S(t)}$ , is the Sudakov factor which suppresses the soft dynamics effectively [12].

The longitudinal and transverse polarization vectors of the axial-vector meson are defined by:

$$\begin{aligned} \epsilon^{*(0)\mu} &= \frac{E}{m_{V(A)}} \left[ \left(1 - \frac{m_{V(A)}^2}{4E^2}\right) n_-^\mu - \frac{m_V^2}{4E^2} n_+^\mu \right], \\ \epsilon_\perp^{*(\lambda)\mu} &= \left( \epsilon^{*(\lambda)\mu} - \frac{\epsilon^{*(\lambda)} n_+}{2} n_-^\mu - \frac{\epsilon^{*(\lambda)} n_-}{2} n_+^\mu \right) \delta_{\lambda, \pm 1}, \end{aligned} \quad (8)$$

where  $n_-^\mu \equiv (1, 0, 0, -1)$  and  $n_+^\mu \equiv (1, 0, 0, 1)$ , in the  $B$  rest frame, for a axial-vector meson is moving along the z-axis, while the  $x$  axes of both daughter particles are parallel the coordinate systems in the Jackson convention [4]:

$$\begin{aligned}\epsilon_1^{\mu(0)} &= (p_c, 0, 0, E_1)/m_1, & \epsilon_2^{\mu(0)} &= (p_c, 0, 0, -E_2)/m_2, \\ \epsilon_1^{\mu(\pm 1)} &= \frac{1}{\sqrt{2}}(0, \pm 1, -i, 0), & \epsilon_2^{\mu(\pm 1)} &= \frac{1}{\sqrt{2}}(0, \mp 1, i, 0),\end{aligned}$$

where  $p_c$  is the center mass momentum of the final state meson and  $\epsilon_1^{*(\pm 1)} \cdot \epsilon_2^{*(\pm 1)} = -\delta_{\pm 1, \pm 1}$ . If the A meson moves along the  $n_-^\mu$ . In the large energy limit, we have  $\epsilon_A^{*(\lambda)} \cdot n_+ = 2E_A/m_A \delta_{\lambda, 0}$  and  $\epsilon_A^{*(\lambda)} \cdot n_- = 0$ . If the coordinate systems are in the Jacob-Wick convention where the y axes of both decay particles are parallel, the transverse polarization vectors of the second meson will become  $\epsilon_2^\mu = (0, \pm 1, -i, 0)/\sqrt{2}$  and  $\epsilon_1^{*(\pm 1)} \cdot \epsilon_2^{*(\pm 1)} = \delta_{\pm 1, \pm 1}$ . For the wave function of  $B_c$  meson, we adopt the form (see Ref [13]. and references therein) as,

$$\Phi_{B_c} = \frac{i}{\sqrt{2N_c}} [(\not{P} + m_{B_c}) \gamma_5 \phi_{B_c}(x)]_{\alpha\beta}. \quad (9)$$

where  $P$  is the momentum of the  $B_c$  meson and  $x$  denotes the momentum fraction of the c quark in the  $B_c$  meson. The distribution amplitude  $B_c$  would be close to  $\delta(x - \frac{m_c}{m_{B_c}})$  in the nonrelativistic limit because  $B_c$  meson embraces b and c quarks simultaneously.

We therefore adopt the non-relativistic approximation form for  $\phi_{B_c}$  as [14, 15],

$$\phi_{B_c} = \frac{f_{B_c}}{2\sqrt{2N_c}} \delta(x - \frac{m_c}{m_{B_c}}), \quad (10)$$

here b is the conjugate space coordinate of transverse momentum  $k_T$ ,  $f_{B_c}$  and  $N_c = 3$  are the decay constant of  $B_c$  meson and the color number respectively. wave function  $\phi_{B_c}$  is normalized by decay constant of  $f_{B_c}$

$$\int_0^1 dx \phi_{B_c}(x, b=0) = \frac{f_{B_c}}{2\sqrt{2N_c}}, \quad (11)$$

For the vector  $J/\psi$  meson(V), we take the wave function as follows,

$$\begin{aligned}\Phi_{J/\psi}^L(x) &= \frac{1}{\sqrt{2N_c}} \{m_{J/\psi} \not{\epsilon}_L \phi^L(x) + \not{\epsilon}_L \not{p} \phi^t(x)\}, \\ \Phi_{J/\psi}^T(x) &= \frac{1}{\sqrt{2N_c}} \{m_{J/\psi} \not{\epsilon}_L \phi^V(x) + \not{\epsilon}_T \not{p} \phi^T(x)\}.\end{aligned} \quad (12)$$

The asymptotic distribution amplitudes of  $J/\psi$  meson read as [16]:

$$\begin{aligned}\phi^L(x) &= \phi^T(x) = 9.58 \frac{f_{J/\psi}}{2\sqrt{2N_c}} x(1-x) \left[ \frac{x(1-x)}{1-2.8x(1-x)} \right]^{0.7}, \\ \phi^t(x) &= 10.94 \frac{f_{J/\psi}}{2\sqrt{2N_c}} (1-2x)^2 \left[ \frac{x(1-x)}{1-2.8x(1-x)} \right]^{0.7}, \\ \phi^V(x) &= 1.67 \frac{f_{J/\psi}}{2\sqrt{2N_c}} [1 + (1+2x)^2] \left[ \frac{x(1-x)}{1-2.8x(1-x)} \right]^{0.7}.\end{aligned} \quad (13)$$

Here,  $\phi^L$  and  $\phi^T$  denote for the twist-2 distribution amplitudes, and  $\phi^t$  and  $\phi^V$  for the twist-3.  $x$  denotes the momentum fraction of the charm quark inside the charmonium. In which the twist-3 ones  $\phi^{t,V}$  vanish, as the twist-2 ones, at the end points due to the factor  $[x(1-x)]^{0.7}$ .

The twist-2 distribution amplitudes for the longitudinally and trasversely polarized axial-vector mesons (A) can be parameterized as[17, 18]:

$$\begin{aligned}\phi_A(x) &= \frac{3f_A}{\sqrt{2N_c}}x(1-x)\{a_{0A}^{\parallel} + 3a_{1A}^{\parallel}(2x-1) + a_{2A}^{\parallel}\frac{3}{2}(5(2x-1)^2-1)\} \\ \phi_A^T(x) &= \frac{3f_A}{\sqrt{2N_c}}x(1-x)\{a_{0A}^{\perp} + 3a_{1A}^{\perp}(2x-1) + a_{2A}^{\perp}\frac{3}{2}(5(2x-1)^2-1)\}\end{aligned}\quad (14)$$

These distribution amplitudes satisfy the relation

$$\begin{aligned}\int_0^1 \phi_{3P_1}(x) &= \frac{f^3 P_1}{2\sqrt{2N_c}}, \\ \int_0^1 \phi_{1P_1}(x) &= \frac{f^1 P_1}{2\sqrt{2N_c}}.\end{aligned}\quad (15)$$

As for twist-3 distribution amplitudes for axial-vector meson, we use the following form

$$\begin{aligned}\phi_A^t(x) &= \frac{3f_A}{2\sqrt{2N_c}}\{a_{0A}^{\perp}(2x-1)^2 + \frac{1}{2}a_{1A}^{\perp}(2x-1)(3(2x-1)^2-1)\}, \\ \phi_A^s(x) &= \frac{3f_A}{2\sqrt{2N_c}}\frac{d}{dx}\{x(1-x)(a_{0A}^{\perp} + a_{1A}^{\perp}0)\}, \\ \phi_A^v(x) &= \frac{3f_A}{4\sqrt{2N_c}}\{\frac{1}{2}a_{0A}^{\parallel}(1+(2x-1)^2) + a_{1A}^{\parallel}(2x-1)^3\}, \\ \phi_A^a(x) &= \frac{3f_A}{4\sqrt{2N_c}}\frac{d}{dx}\{x(1-x)(a_{0A}^{\parallel} + a_{1A}^{\parallel}(2x-1))\}.\end{aligned}\quad (16)$$

Here  $x$  represents the momentum fraction carried by quark in the meson and  $a_{03P_1}^{\parallel} = 1$ ,  $a_{03P_1}^{\perp} = 0$ .

The intrinsic  $b$  dependence for the heavy meson wave function  $\phi_{B_c}$  is important and for the light axial-vector meson wave function  $\phi_A$  is negligible[19].

### 3 Branching ratio of $B_c^+ \rightarrow J/\psi a_1(1260)^+$ decay

In the pQCD approach, the four Feynman diagrams for  $B_c^+ \rightarrow J/\psi a_1(1260)^+$  decays are shown in Fig.1. There are three kinds of polarizations of a axial-vector meson, namely, longitudinal ( $L$ ), normal ( $N$ ), and transverse ( $T$ ). The decay amplitudes for the factorizable diagram (a) and (b) can be read as,

(i)  $(V-A)(V-A)$

$$\begin{aligned}F_{fs}^L &= 8\pi C_F f_A m_{B_c}^2 \int_0^1 dx_1 dx_3 \int_0^\infty b_1 db_1 b_3 db_3 \phi_{B_c}(x_1, b_1) \\ &\quad \times \{[(1+x_3)\phi^V(x_3) + r_V(1-2x_3)(\phi^L(x_3) + \phi^t(x_3))]\} \\ &\quad \times E_{fs}(t_a) h_{fs}(x_1, x_3, b_1, b_3) + 2r_V \phi^T(x_3) \\ &\quad \times E_{fs}(t_b) h_{fs}(x_3, x_1, b_3, b_1)\},\end{aligned}\quad (17)$$

where  $C_F = 4/3$  is a color factor,  $r_i = m_i/m_B$ , and  $f_A$  is the decay constant of  $a_1$  meson. The convolution functions  $E_{ef}$ , the hard function  $h_{ef}$  and the factorization hard scal  $t_{a,b}$  are given in the appendix.

(ii)  $(V - A)(V + A)$

$$F_{fs}^{L;P_1} = -F_{fs}^L, \quad (18)$$

which is originated from  $\langle a_1 | V + A | 0 \rangle = -\langle a_1 | V - A | 0 \rangle$ .

(iii)  $(S - P)(S + P)$

$$F_{fs}^{L;P_2} = 0, \quad (19)$$

because the emitted axial-vector meson can not be produced through a scalar or a pseudoscalar current.

For the non-factorizable diagrams (c) and (d), the decay amplitude can be read as

(i)  $(V - A)(V - A)$

$$\begin{aligned} M_{nfs}^L = & \frac{32}{\sqrt{6}} \pi C_F m_{B_c}^2 \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \phi_{B_c}(x_1, b_1) \\ & \times \phi_A(x_2) \{ [(1 - x_2) \phi^V(x_3) - r_V x_3 (\phi^L(x_3) - \phi^t(x_3))] \\ & \times E_{nfs}(t_c) h_{nfs}(x_1, 1 - x_2, x_3, b_1, b_2) - [(x_2 + x_3) \phi^V(x_3) \\ & - r_V x_3 (\phi^V(x_3) - r_V x_3 (\phi^T(x_3) + \phi^t(x_3)))] \\ & \times E_{nfs}(t_d) h_{nfs}(x_1, x_2, x_3, b_1, b_2) \}, \end{aligned} \quad (20)$$

(ii)  $(V - A)(V + A)$

$$\begin{aligned} M_{nfs}^{L;P_1} = & \frac{32}{\sqrt{6}} \pi C_F m_{B_c}^2 \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \phi_{B_c}(x_1, b_1) \\ & \times r_V \{ [(1 - x_2) (\phi_A^s(x_2) + \phi_A^t(x_2)) \phi^V(x_3) - r_V (\phi_A^s(x_2) \\ & \times [(x_2 - x_3 - 1) \phi^L(x_3) - (x_2 + x_3 - 1) \phi^t(x_3)] + \phi_A^t(x_2) \\ & \times [(x_2 + x_3 - 1) \phi^T(x_3) + (1 - x_2 + x_3) \phi^t(x_3)] \\ & \times E_{nfs}(t_c) h_{nfs}(x_1, 1 - x_2, x_3, b_1, b_2) - [x_2 (\phi_A^s(x_2) \\ & - \phi_A^t(x_2)) \phi^V + r_V (x_2 (\phi_A^s(x_2) - \phi_A^t(x_2)) (\phi^L(x_3) - \phi^t(x_3)) \\ & + x_3 (\phi_A^s(x_2) + \phi_A^t(x_2)) (\phi^V(x_3) + \phi^t(x_3)))] \\ & \times E_{nfs}(t_d) h_{nfs}(x_1, x_2, x_3, b_1, b_2) \}, \end{aligned} \quad (21)$$

(iii)  $(S - P)(S + P)$

$$\begin{aligned} M_{nfs}^{L;P_2} = & \frac{32}{\sqrt{6}} \pi C_F m_{B_c}^2 \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \phi_{B_c}(x_1, b_1) \\ & \times \phi_A(x_2) \{ [(x_2 - x_3 - 1) \phi^V(x_3) + r_V x_3 (\phi^L(x_3) \\ & + \phi^t(x_3))] E_{nfs}(t_c) h_{enf}(x_1, 1 - x_2, x_3, b_1, b_2) \\ & + [x_2 \phi^V(x_3) - r_V x_3 (\phi^L(x_3) - \phi^t(x_3))] \\ & \times E_{nfs}(t_d) h_{enf}(x_1, x_2, x_3, b_1, b_2) \}, \end{aligned} \quad (22)$$

We can also present the factorization formulas for the Feynman amplitudes with trans-verse polarizations,

$$\begin{aligned}
F_{fs}^N &= 8\pi C_F f_A m_{B_c}^2 \int_0^1 dx_1 dx_3 \int_0^\infty b_1 db_1 b_3 db_3 \phi_{B_c}(x_1, b_1) r_V \\
&\quad \times \{ [\phi^t(x_3) + r_V x_3 (\phi^V(x_3) - \phi^L(x_3)) + 2r_V \phi^V(x_3)] \\
&\quad \times E_{fs}(t_a) h_{fs}(x_1, x_3, b_1, b_3) + r_V (\phi^T(x_3) + \phi^V(x_3)) \\
&\quad \times E_{fs}(t_b) h_{fs}(x_3, x_1, b_3, b_1) \}, \tag{23}
\end{aligned}$$

$$\begin{aligned}
F_{fs}^T &= 16\pi C_F f_A m_{B_c}^2 \int_0^1 dx_1 dx_3 \int_0^\infty b_1 db_1 b_3 db_3 \phi_{B_c}(x_1, b_1) r_V \\
&\quad \times \{ [\phi^t(x_3) - r_V x_3 (\phi^V(x_3) - \phi^L(x_3)) + 2r_V \phi^T(x_3)] \\
&\quad \times E_{fs}(t_a) h_{fs}(x_1, x_3, b_1, b_3) + r_V (\phi^L(x_3) + \phi^V(x_3)) \\
&\quad \times E_{fs}(t_b) h_{fs}(x_3, x_1, b_3, b_1) \}, \tag{24}
\end{aligned}$$

$$\begin{aligned}
F_{fs}^{N;P_1} &= -F_{fs}^N, \\
F_{fs}^{T;P_1} &= -F_{fs}^T,
\end{aligned}$$

$$\begin{aligned}
F_{fs}^{N;P_2} &= 0, \\
F_{fs}^{T;P_1} &= 0,
\end{aligned}$$

$$\begin{aligned}
M_{nfs}^N &= \frac{32}{\sqrt{6}} \pi C_F m_{B_c}^2 \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \phi_{B_c}(x_1, b_1) r_A \\
&\quad \times \{ [(1-x_2)(\phi_A^a(x_2) + \phi_A^v(x_2))\phi^t(x_3)] h_{nfs}(x_1, 1-x_2, x_3, b_1, b_2) \\
&\quad \times E_{nfs}(t_c) + [x_2(\phi_A^a(x_2) + \phi_A^v(x_2))\phi^t(x_3) - 2r_V(x_2+x_3) \\
&\quad \times (\phi_A^a(x_2)\phi_V^a(x_3) + \phi_A^v(x_2))\phi^t(x_3)] E_{nfs}(t_d) h_{nfs}(x_1, x_2, x_3, b_1, b_2) \}, \tag{25}
\end{aligned}$$

$$\begin{aligned}
M_{nfs}^T &= \frac{64}{\sqrt{6}} \pi C_F m_{B_c}^2 \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \phi_{B_c}(x_1, b_1) r_A \\
&\quad \times \{ [(1-x_2)(\phi_A^a(x_2) + \phi_A^v(x_2))\phi^t(x_3)] E_{nfs}(t_c) \\
&\quad \times h_{nfs}(x_1, 1-x_2, x_3, b_1, b_2) + [x_2(\phi_A^a(x_2) + \phi_A^v(x_2)) \\
&\quad \times \phi^t(x_3) - 2r_V(x_2+x_3)(\phi_A^a(x_2)\phi^L(x_3) + \phi_A^v(x_2))] \\
&\quad \times E_{nfs}(t_d) h_{nfs}(x_1, x_2, x_3, b_1, b_2) \}, \tag{26}
\end{aligned}$$

$$\begin{aligned}
M_{nfs}^{N;P_1} &= -\frac{32}{\sqrt{6}} \pi C_F m_{B_c}^2 \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \phi_{B_c}(x_1, b_1) r_A \\
&\quad \times x_3 \phi_A^T(x_2) (\phi^L(x_3) - \phi^V(x_3)) [E_{nfs}(t_c) h_{nfs}(x_1, 1-x_2, x_3, b_1, b_2) \\
&\quad + E_{nfs}(t_d) h_{nfs}(x_1, x_2, x_3, b_1, b_2)], \tag{27}
\end{aligned}$$

$$\begin{aligned}
M_{nfs}^{T;P_1} &= -\frac{64}{\sqrt{6}} \pi C_F m_{B_c}^2 \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \phi_{B_c}(x_1, b_1) r_A \\
&\quad \times x_3 \phi_A^T(x_2) (\phi^L(x_3) - \phi^V(x_3)) [E_{nfs}(t_c) h_{nfs}(x_1, 1-x_2, x_3, b_1, b_2) \\
&\quad + E_{nfs}(t_d) h_{nfs}(x_1, x_2, x_3, b_1, b_2)], \tag{28}
\end{aligned}$$

$$\begin{aligned}
M_{nfs}^{N;P_2} = & -\frac{32}{\sqrt{6}}\pi C_F m_{B_c}^2 \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \phi_{B_c}(x_1, b_1) r_A \\
& \times \{ [x_2(\phi_A^a(x_2) - \phi_A^v(x_2))\phi^t(x_3)] E_{nfs}(t_d) h_{nfs}(x_1, 1-x_2, x_3, b_1, b_2) \\
& \times [x_2(\phi_A^a(x_2) - \phi_A^v(x_2))\phi^t(x_3) + 2r_V(1-x_2+x_3)(\phi_A^v(x_2)\phi^V(x_3) \\
& - \phi_A^a(x_2)\phi^L(x_3))] E_{nfs}(t_c) h_{nfs}(x_1, x_2, x_3, b_1, b_2) \}, \quad (29)
\end{aligned}$$

$$\begin{aligned}
M_{nfs}^{T;P_2} = & -\frac{64}{\sqrt{6}}\pi C_F m_{B_c}^2 \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \phi_{B_c}(x_1, b_1) r_A \\
& \times \{ [x_2(\phi_A^a(x_2) - \phi_A^v(x_2))\phi^t(x_3)] h_{nfs}(x_1, 1-x_2, x_3, b_1, b_2) \\
& \times E_{nfs}(t_d) + [x_2(\phi_A^a(x_2) - \phi_A^v(x_2))\phi^t(x_3) + 2r_V(1-x_2+x_3) \\
& \times (\phi_A^v(x_2)\phi^V(x_3)\phi_A^a(x_2)\phi^L(x_3))] h_{nfs}(x_1, x_2, x_3, b_1, b_2) E_{nfs}(t_c) \} \quad (30)
\end{aligned}$$

Decay amplitudes with three polarizations  $h = L, N, T$  as follows,

$$M = V_{cb} V_{ud}^* [(C_1 + \frac{C_2}{3}) F_{fs}^h + C_2 M_{nfs}^h], \quad (31)$$

the decay rate can be written explicitly as

$$\Gamma = \frac{G_F^2 P}{16\pi m_{B_c}^2} \sum_{\sigma=L,N,T} M^{\sigma\dagger} M^\sigma, \quad (32)$$

where  $G_F$  is the Fermi coupling constant.  $V_{cb}$  and  $V_{ud}$  are the Cabibbo-Kobayashi-Maskawa (CKM) matrix factors and  $C_i$  are the Wilson coefficients. The branching ratio is given by

$$BR = \Gamma \times \tau_{B_c}. \quad (33)$$

The decay amplitudes  $M^\sigma$  can be described by

$$\begin{aligned}
M^\sigma = & \epsilon_{2\mu}^*(\sigma) \epsilon_{3\nu}^*(\sigma) [a g^{\mu\nu} + \frac{b}{m_2 m_3} P_{B_c}^\mu P_{B_c}^\nu + i \frac{c}{m_2 m_3} \epsilon^{\mu\nu\alpha\beta} P_2^\alpha P_3^\beta] \\
= & M_L + M_N \epsilon_2^*(\sigma = T) \cdot \epsilon_3^*(\sigma = T) + i \frac{M_T}{m_{B_c}^2} \epsilon^{\alpha\beta\gamma\rho} \epsilon_{2\alpha}^*(\sigma) \epsilon_{2\beta}^*(\sigma) P_{2\gamma} P_{3\rho}, \quad (34)
\end{aligned}$$

the subscript  $\sigma$  denotes the helicity states of the two final mesons. The amplitude  $M^\sigma$  can be decomposed, according to the Lorentz-invariant amplitudes a, b and c [20]

$$\begin{aligned}
m_{B_c}^2 M_L = & a \epsilon_2^*(L) \cdot \epsilon_3^*(L) + \frac{b}{m_2 m_3} \epsilon_2^*(L) \cdot P_3 \epsilon_2^*(L) \cdot P_2, \\
m_{B_c}^2 M_N = & a, \\
m_{B_c}^2 M_T = & \frac{m_{B_c}^2}{m_2 m_3} c. \quad (35)
\end{aligned}$$

We can define the amplitudes  $A_{(i=L,\parallel,\perp)}$  as

$$\begin{aligned}
A_L = & -\xi m_{B_c}^2 M_L, \\
A_\parallel = & \xi \sqrt{2} m_{B_c}^2 M_N, \\
A_\perp = & \xi r_2 r_3 \sqrt{2(r^2 - 1)} m_{B_c}^2 M_T, \quad (36)
\end{aligned}$$



for the longitudinal, parallel, and perpendicular polarizations, respectively, with the normalization factor  $\xi = \sqrt{G_F^2 p_c / (16\pi m_{B_c}^2 \Gamma)}$  and  $r = P_2 P_3 / (m_2 m_3)$ . These amplitudes satisfy the relation,

$$|A_L|^2 + |A_{\parallel}|^2 + |A_{\perp}|^2 = 1. \quad (37)$$

The polarization fractions  $f_L$ ,  $f_{\parallel}$  and  $f_{\perp}$  can be defined as

$$f_{L(\parallel, \perp)} = \frac{|A_{L(\parallel, \perp)}|^2}{|A_L|^2 + |A_{\parallel}|^2 + |A_{\perp}|^2} = |A_{L(\parallel, \perp)}|^2. \quad (38)$$

The direct CP asymmetry  $A_{cp}^{dir}$  is defined as [21]

$$A_{cp}^{dir, \alpha} = \frac{\bar{f}_{\alpha} - f_{\alpha}}{\bar{f}_{\alpha} + f_{\alpha}} (\alpha = L, \parallel, \perp). \quad (39)$$

## 4 Numerical results

We use other input parameters as follows:

$m_{B_c} = 6.277 \text{ GeV}$ ,  $m_{a_1} = 1.23 \text{ GeV}$ ,  $m_j/\psi = 3.096 \text{ GeV}$ ,  $m_c = 1.27 \text{ GeV}$ ,  $m_W = 80.4 \text{ GeV}$ ,  $f_{B_c} = (0.489 \pm 0.004) \text{ GeV}$ ,  $f_{j/\psi} = (0.405 \pm 0.014) \text{ GeV}$ ,  $f_{a_1} = 0.238 \text{ GeV}$ ,  $a_2^{\parallel, a_1} = -0.02 \pm 0.02$ ,  $a_1^{\perp, a_1} = -1.04 \pm 0.34$ ,  $\tau_{B_c} = 0.46 \text{ ps}$ ,  $\Lambda_{QCD} = 0.2250 \text{ GeV}$ ,  $G_F = 1.16 \times 10^{-5}$  [3, 22, 23, 24]

The CKM matrix is a  $3 \times 3$  unitary matrix as [22]

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

The elements of the CKM matrix can be parameterized by three mixing angles  $A, \lambda, \rho$  and a CP-violating phase  $\eta$

$$V = \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ \lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

The results for the Wolfenstein parameters are

$$\lambda = 0.22535 \pm 0.00065, \quad A = 0.811_{-0.012}^{+0.022}, \\ \bar{\rho} = 0.131_{-0.013}^{+0.026}, \quad \bar{\eta} = 0.345_{-0.014}^{+0.013}.$$

We use the central values of the Wolfenstein parameters and obtain

$$V_{cb} = 0.0412_{-0.0005}^{+0.0011}, \quad V_{ud} = 0.97427 \pm 0.00015.$$

The Wilson coefficients  $C_1$  and  $C_2$ , the coupling constants for the interaction terms  $Q_1$  and  $Q_2$ , become calculable nontrivial functions of  $\alpha_s$ ,  $M_W$  and the renormalization scale  $\mu$ . Wilson coefficients  $C(\mu_W)$  at  $\mu_W = M_W$  are  $C_1(\mu_W) = 0$ ,  $C_2(\mu_W) = 1$  [25]. In the generalized factorization approach, it is well established that non-factorizable contributions must be present in the

matrix elements in order to cancel the scale  $\mu$  and the renormalization scheme dependence of  $C_i(\mu)$ . To solve the issue of scale  $\mu$  dependence, but not the renormalization scheme dependence, to isolate form the matrix element of four operators the dependence, and link with the  $\mu$  dependence in the Wilson coefficients  $C_i(\mu)$ . In table (1)[26], we simply present the numerical results of the leading order (LO) and next-to-leading order (NLO) Wilson coefficients in different scales. We use the NLO Wilson coefficient because NLO contributions in the renormalization group improved perturbation theory.

Table 1: The values of LO and NLO Wilson coefficients  $C_i(\mu)$  for  $\mu = m_b$ ,  $\mu = m_b/2$ ,  $\mu = 2m_b$

	$\mu = m_b/2$		$\mu = m_b$		$\mu = 2m_b$	
	LO	NLO	LO	NLO	LO	NLO
$C_1$	1.179	1.115	1.179	1.080	1.072	1.043
$C_2$	-0.370	-0.280	-0.255	-0.180	-0.171	-0.104

The branching ratio of  $B_c^+ \rightarrow J/\psi a_1(1260)^+$  has been calculated at three scales, e.g.  $\mu = m_b$ ,  $\mu = m_b/2$  and  $\mu = 2m_b$  in table (2). The experimental data of branching ratio of this decay is less than  $1.3 \times 10^{-3}$  [22], the branching at the scale  $\mu = m_b$  is nearly agreement with the experimental result.

For the theoretical uncertainties in our calculation, we estimated three kinds

Table 2: Branching ratios of the  $B_c^+ \rightarrow J/\psi a_1(1260)^+$  decay

energy scale	Branching ratios
$\mu = m_b/2$	$(0.97^{+0.05+0.03+0.02}_{-0.06-0.02-0.01}) \times 10^{-3}$
$\mu = m_b$	$(1.02^{+0.04+0.03+0.01}_{-0.08-0.05-0.01}) \times 10^{-3}$
$\mu = 2m_b$	$(1.3^{+0.06+0.02+0.01}_{-0.07-0.04-0.03}) \times 10^{-3}$

of them: The first errors in our calculations are caused by the the decay constants, the shape parameters in wave function  $B_c$ , Gegenbauer moments of  $a_1$  and the hard energy scale  $\mu_i$ ; The second errors are from the unknown next-to-leading order QCD corrections with respect to  $\alpha_s$  and the power corrections, characterized by the choice of the  $\Lambda_{QCD}$ ; The third error are estimated from the uncertainties of the CKM matrix elements.

We calculate the direct CP-violating asymmetry in every polarization and we obtain the results in the pQCD approach as

$$A_{cp}^{dir,L} \approx 0.0, A_{cp}^{dir,\parallel} \approx 0.0, A_{cp}^{dir,\perp} \approx 0.0.$$

There is only one kind of Cabibbo-Kabayashi-Muskawa (CKM) phase involved in the decay amplitude (see Eq.(31)), therefore CP violation is absent for every polarization in this decay.

## 5 conclusion

In this work we have presented a detailed study of two-body  $B_c$  decays into final states involving one vector and one axial-vector meson (VA), within the

framework of pQCD approach. We have calculated the branching ratio of  $B_c^+ \rightarrow J/\psi a_1(1260)^+$  decay in pQCD approach based on the  $k_T$  factorization theorem. We calculated not only the factorizable emission diagrams, but also the nonfactorizable spectator. The experimental data of branching ratio of this decay is lesser than  $(1.3 \times 10^{-3})$ . We obtained the branching ratio of pQCD approach is  $((1.02_{-0.08-0.05-0.01}^{+0.04+0.03+0.01}) \times 10^{-3})$  at scale  $\mu = m_b$ , this branching ratio has an agreement with the experimental result. Our theoretical predictions in the pQCD approach will provide an important platform for testing the SM and exploring the helicity structure of this considered decay. It can also provide more information on measuring the unitary CKM angles and understanding the decay mechanism.

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## A Appendix

The function  $E_{fs}$  and  $E_{nfs}$  defined as

$$\begin{aligned} E_{fs}(t) &= \alpha_s(t).exp[-S_{B_c}(t) - S_2(t)] \\ E_{nfs}(t) &= \alpha_s(t).exp[-S_2(t) - S_3(t)]. \end{aligned}$$

The Sudakov exponents are defined as

$$\begin{aligned} S_{B_c}(t) &= s(x_1 \frac{m_{B_c}}{\sqrt{2}}, b_1) + \frac{5}{3} \int_{1/b_1}^t \frac{d\bar{\mu}}{\bar{\mu}} \gamma_q(\alpha_s(\bar{\mu})), \\ S_2(t) &= s(x_2 \frac{m_{B_c}}{\sqrt{2}}, b_2) + s((1-x_2) \frac{m_{B_c}}{\sqrt{2}}, b_2) + 2 \int_{1/b_2}^t \frac{d\bar{\mu}}{\bar{\mu}} \gamma_q(\alpha_s(\bar{\mu})), \\ S_3(t) &= s(x_3 \frac{m_{B_c}}{\sqrt{2}}, b_3) + s((1-x_3) \frac{m_{B_c}}{\sqrt{2}}, b_3) + 2 \int_{1/b_3}^t \frac{d\bar{\mu}}{\bar{\mu}} \gamma_q(\alpha_s(\bar{\mu})), \\ s(Q, b) &= \int_{1/b}^Q \frac{d\bar{\mu}}{\bar{\mu}} [\{ \frac{2}{3}(2\gamma_E - 1 - \log 2) + C_F \log \frac{d\bar{\mu}}{\bar{\mu}} \} \frac{\alpha_s}{\pi} \\ &\quad + \{ \frac{67}{9} - \frac{\pi^2}{3} + \frac{10}{27} n_f + \frac{2}{3} \beta_0 \log \frac{\gamma_E}{2} \} (\frac{\alpha_s(\bar{\mu})}{\pi})^2 \log \frac{Q}{\bar{\mu}}] \end{aligned}$$

where the function  $s(Q, b)$  are defined in the Appendix A of Ref [27].  
 Here

$$\begin{aligned} t_a &= \max[\sqrt{x_3} m_{B_c}, 1/b_2, 1/b_3], \\ t_b &= \max[\sqrt{x_2} m_{B_c}, 1/b_2, 1/b_3], \\ t_c &= \max[\sqrt{x_1 x_3} m_{B_c}, \sqrt{(x_1 - x_2) x_3} m_{B_c}, 1/b_1, 1/b_2], \\ t_d &= \max[\sqrt{x_2 x_3} m_{B_c}, \sqrt{|x_1 - x_2| x_3} m_{B_c}, 1/b_1, 1/b_2], \end{aligned}$$

$$\begin{aligned} h_{fs}(x_1, x_2, x_3, b_2, b_3) &= (\frac{i\pi}{2})^2 H_0^{(1)}(\sqrt{x_2 x_3} m_{B_c} b_2) \\ &\quad \times [(\theta(b_2 - b_3)) H_0^{(1)}(\sqrt{x_3} m_{B_c} b_2) J_0(\sqrt{x_3} m_{B_c} b_3) \\ &\quad + \theta(b_3 - b_2)) H_0^{(1)}(\sqrt{x_1 + x_2 + x_3 + x_3 x_1} m_{B_c} b_3) \\ &\quad \times J_0(\sqrt{x_3} m_{B_c} b_2)]. S_t(x_3) \end{aligned}$$

The threshold resummation form factor  $S_t(x_i)$  is adopted from Ref [28].

$$S_t(x) = \frac{2^{l+2c}\Gamma(3/2+c)}{\sqrt{\pi}\Gamma(1+c)}[x(1-x)]^c,$$

with  $c = 0.3$  in this work.

$$\begin{aligned} h_{nfs}(x_1, x_2, x_3, b_1, b_2) &= \frac{i\pi}{2} [(\theta(b_1 - b_2)H_0^{(1)}(\sqrt{x_2x_3}m_{B_c}b_1)J_0^{(1)}(\sqrt{x_2x_3}m_{B_c}b_2) \\ &\times \frac{\frac{i\pi}{2}H_0^{(1)}(\sqrt{|F_j^2|m_B}b_1)}{K_0(F_jm_Bb_1)} \quad \frac{F_j^2\langle 0}{F_j^2\langle 0} \end{aligned}$$

$$\begin{aligned} F_1^2 &= 1 - (1 - x_3)(1 - x_1 - x_2), \\ F_1^2 &= x_3(x_1 - x_2). \end{aligned}$$

And

$$H_0^{(1)}(z) = J_0(z) + iK_0(z)$$